Acoustic Wave Driven MTF Fusion Reactor

By

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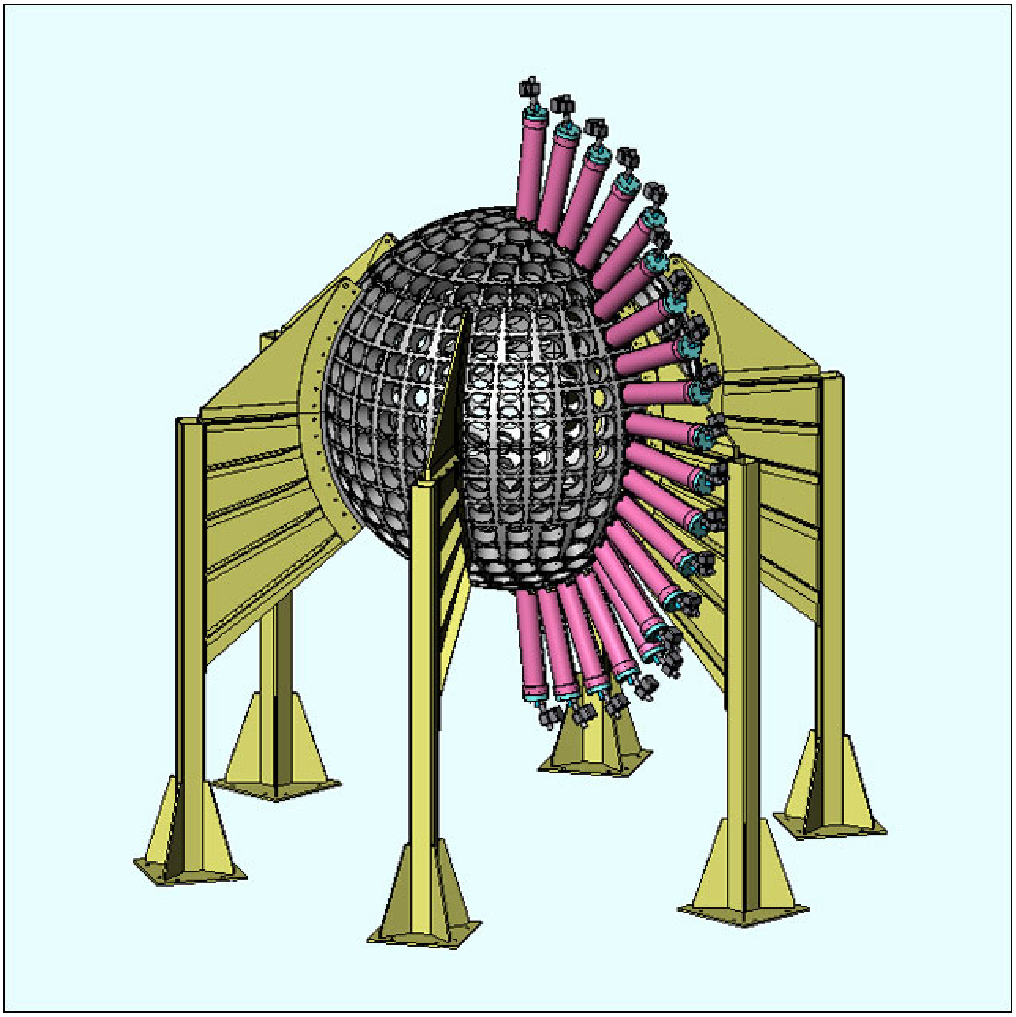
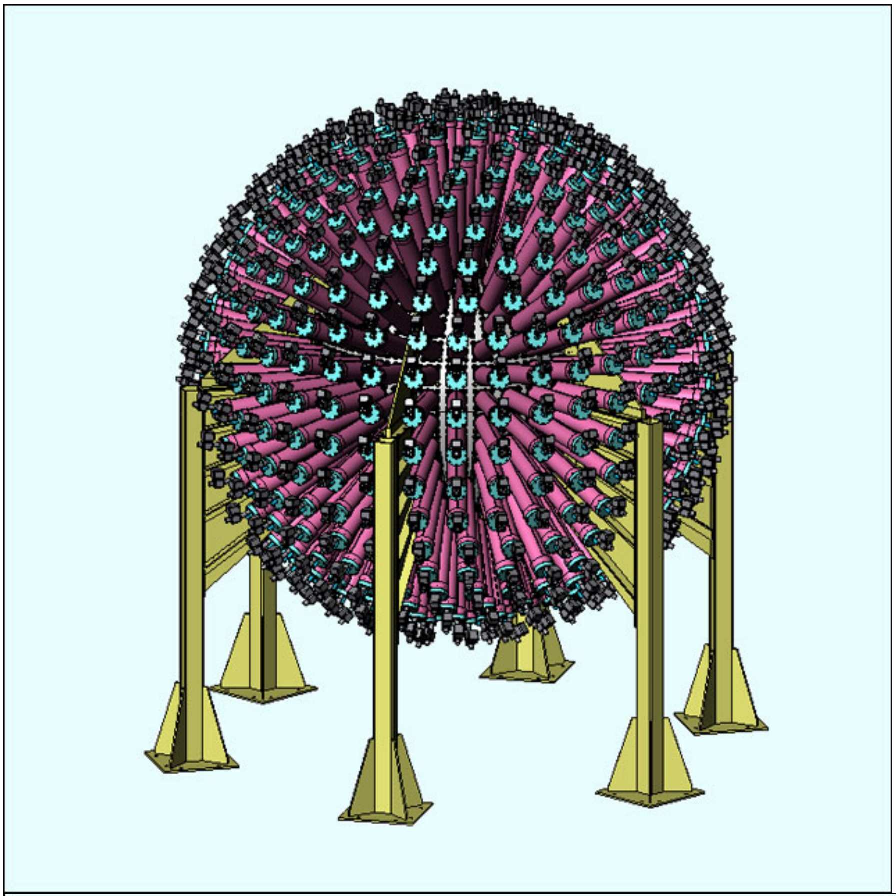
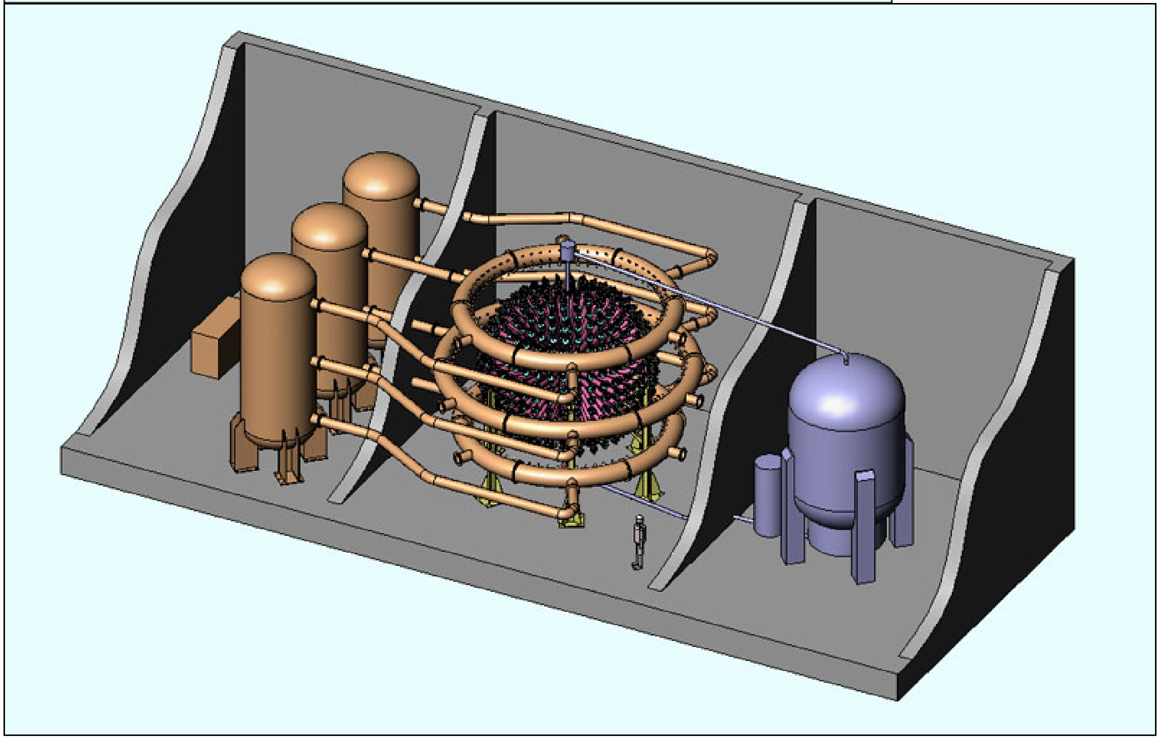
Magnetized Target Fusion (MTF) is an interesting fusion regime that is presently being investigated my many researchers. The basic idea is to rapidly compress a pre-formed magnetized plasma configuration to thermonuclear temperature and pressure. The magnetic field provides some level of confinement and thermal insulation of the plasma. Many methods have been proposed to accomplish the compression. Most research is focused on the deuterium-tritium reaction as it is the one requiring the least drastic conditions. We are also considering this fuel.

We propose a new compression system that offers many advantages. A near spherical vessel ~2 m in diameter is filled with liquid lithium-lead alloy (Li-Pb). This liquid is under consideration for fusion reactor blanket; it has low melting point, low vapor pressure, re-breeds the tritium, and good nuclear characteristics. The liquid is spun in the vessel by pumps that inject the liquid tangentially near the equator and pumps it out near the poles. This creates a vertical vortex tube in the liquid metal. The vessel is surrounded by many steam actuated pistons. The steam accelerates the pistons to ~100 m/s. The pistons impact the spherical vessel and send a strong acoustic wave in the liquid metal. The pressure developed at the impact is: $P=\rho v c_s/2$ where ρ is the density, v the speed of impact and c_s is the sound speed in the impacting material. For steel ρ=8000 kg/m and c_s=5000 m/s so the pressure developed is 2 GPa. Good steel can handle up to 450 kpsi (3 GPa) of compression. The efficiency of the driver can be quite good. About 33% of the thermal energy goes into piston kinetic energy (the usual thermal to mechanical efficiency at realistic temperature). For steel and liquid lead (density 10.8, c_s= 2 km/s), the acoustic impedance (density*speed of sound) match is good with 91% of the energy going into the liquid lead. The wave then focuses in the center, getting stronger. Just prior to the wave collapsing the center vortex, two spheromaks (a toroidal magnetized plasma configuration) of reverse helicity are injected from the top and bottom of the system. They move rapidly to the center where they merge to produce a stationary FRC (Field Reverse Configuration). The advantages of this plasma target are that it can be rapidly sent in the center just prior to collapse and then stays there with low velocity while the vortex collapses and compresses it. Also, it has been observed [1] that when merging, the resulting plasma has higher ion temperature than electron temperature. As radiation losses increase with electron temperature but fusion goes with ion temperature, this may somewhat improve the operation. After compression, the fusion energy is released in neutrons that heat the liquid metal. The cycle is repeated at ~1 Hz. The liquid metal goes in a heat exchanger to make steam. The steam is directly used to push on the pistons. Therefore the re-circulated power does not have to be converted in electricity, reducing the cost of the turbo-machinery and generator. Typical MTF systems use pulse power technology worth around 3$/J. For typical fusion systems of order 100 MJ this is $300 million just for the pulse power system. 100 MJ of steam at 1500 psi in a 10 m^3 tank plus associated fast acting valves will cost of the order of $500 000; a considerable saving. Because of the high accuracy of the impact timing of the numerous pistons (~1 μs), an electric means of controlling the exact piston trajectory is required. But this system can control only a few % of the piston energy. In particular, it can be a breaking only system not requiring any high electrical power components. The pistons are sent a few percent above the required velocity and a servo loop applies just the required breaking to adjust the impact time and velocity. The US patent application #11/072,963 describes such a system in more details. The spheromak generator will use pulse power electrical system. But as only ~1% of the compression energy is required for the initial plasma, this should be only a 1 MJ system worth ~3 M$. Most neutrons and all other radiations are stopped in the ~1 m radius of Li-Pb so the neutron flux at the wall is much reduced. This is extremely advantageous over many other fusion systems where neutron and radiation wall loading is a difficult and mostly unresolved technical issue. Radio isotopes produced by neutron activation of the structure is also a problem, especially for maintenance, in most proposed fusion machines. Expensive robotic maintenance is the usual answer to this problem. It is much less of an issue for our proposed machine. Many MTF systems under consideration also require the destruction and replacement of substantial amounts of hardware for each pulse; a costly and complex proposition. Our proposal does not require hardware replacement for each pulse.







This view is for a 3 meter diameter tank without the plasma injector, heat exchanger or turbo-machinery. The 3 tanks on the left store the high pressure steam. The tank on the right is for melting and holding the liquid metal.

We sent the parts for a quotation to a manufacturer, and the cost for these components for a machine of this size came in at 8 M$ Canadian.

Repetition rate operation in the 1 Hz region should not be a problem for the acoustic driver. The main limitation on the repetition rate will be the time for the vortex to re-form to sufficient accuracy after being hit. This could actually be quite long. Notice that a system that measures the vortex (with ultrasound for example) and adjusts the timing of all the pistons to achieve a nice collapse, even for imperfect initial symmetry, could be used to speed up the firing rate. Some magnetic field may be useful to damp out some of the wave in the liquid metal.

Assuming a repetition rate of 0.5 Hz, 100 MJ acoustic pulses, a fusion gain of 6 (relative to impact energy) and a thermal to mechanical efficiency of 33%; this machine would produce ~50 MW electric. Such a small size would reduce time and cost of development compared to the ~1GW fusion power plant usually proposed.

This idea is similar to the linus concept, but can generate substantially faster collapse velocity in the 2.5 km/s range. Faster collapse reduces the confinement quality required of the plasma. This being a big problem; faster is better.

We believe that such a machine, if the difficult fusion in the center can be achieved with sufficient energy gain (a big if), could be built and operated economically. This is not the case for many other fusion approaches. One may not appreciate the use of what amounts to a large reciprocating steam engine as a fusion driver. But if you look at any present day industrial machinery; it is all pipes, pistons, valves, turbines, etc. Not many superconductor magnets, MJ lasers, particle beams and 100 MJ pulse power systems can be found in wide spread industrial equipment out there. Just look under the hood of your car.

Computer simulation

To have a rough idea of the feasibility of this method, we tried to have a computer simulation done by leading fusion research center. We did not manage to produce enough interest to have them run their available codes on this problem. We also looked into commercial hydro-codes but it is hard to know exactly what physics is in the code, they are rather costly and our application is somewhat extreme. So we decided to write our own simple code.

Euler code (fixed cells with liquid flowing through) was not good at dealing with the free surface of the liquid being shot out when the acoustic pressure arrives at the liquid metal-plasma surface. So we used a Lagrange code (cells move with the fluid). We used a 1D spherical (not cylindrical) code. This reduces the radial compression required to increase the pressure to the required value, therefore reducing the accuracy of the implosion. We used pure liquid lead.

The position and velocity of the boundaries between cells are tracked. The pressure, density and temperature are defined in the center of each cell. Linear extrapolation is used to find the value of these parameters at the boundaries between cells. As a cell moves toward the center, it gets thicker. This generated a problem where fusion yield was a function of cell thickness. Thick cells also create non physical oscillation of the plasma density at maximum compression. You need cells much smaller than the minimum size of the plasma at maximum compression, to avoid this dependence. Unfortunately that gives cells so small at the start of the compression that numerical issues arise. We used a cell doubling scheme to fix that problem. As cells move towards the center, they double when they become twice as thick.

Artificial damping is used to prevent numerical instability. We used the minimum possible, as tested with the code, for various times. When the acoustic pulse moves in the liquid; very little damping is required. If the pulse develops into a shock wave; a lot more damping is used to prevent unphysical oscillation at the shock. The damping is tuned so there is only a small speed overshoot at the top of the shock. The energy damped out is put into thermal energy. The total energy is checked and is conserved to a few %. When a shock occurs, we checked the post-shock condition from the code and found them in good agreement with the Hugoniot shock equations. It is actually neat to see how the damping adjusts everything just right to conserve the energy and momentum. The shock is very dissipative and it is best to adjust the acoustic pulse shape to delay the formation of the shock as much as possible. Shock formation and dissipation is very important in this scheme and limits the power that can be delivered at the plasma. Energy can always be increased using a bigger sphere, but the pulse length needs to be increased to delay shock formation and limit losses.

After the wave break out in the plasma cavity, the liquid shoots towards the center. Care must be used here not to put too much damping. The speed distribution curvature is positive as the cells near the center are rapidly accelerating. Damping increases energy in positive speed curvature; this can easily increase the fusion yield un-physically. The yield is a fast function of the energy in the system. The total energy is accounted for and the damping adjusted so that the oscillations remain within reason and the energy does not increase by more than 1%. This is why there is some oscillations there, it could be damped more but that increases energy too much and affects fusion yield.

The liquid accelerates to supersonic speed of ~2.5 km/s and then is suddenly stopped when the plasma pressure rapidly rises at maximum compression. This sends a strong shock outwards that reverse the direction of the fluid. A large amount of damping is required to prevent oscillation of this strong shock. Again, the dissipated energy is put into thermal energy. The conditions before and after the shock are also in good agreement with the Hugoniot shock equation.

The equation of state for the lead was found in [2, 3]. The papers state that they are good up to 10 Mbar. The pressure in the simulation goes to 5 Mbar, well within this limit.

The speed of sound in the preformed plasma is ~100 km/s, much faster than the liquid velocity. The Alven velocity is ~200 km/s, also very fast. These speeds increase as the plasma is compressed. We therefore assume a plasma in equilibrium at all times (no wave equation in the plasma). The plasma is simply assumed to be of uniform density and temperature filling all the volume in the cavity. A magnetic field that starts at $\beta=0.1$ is assumed to be in the plasma and compress with it: $B=B_0/r^2$. The energy in the plasma is increased by the compression energy. We put in Bremsstrahlung radiation loses for Z=1. That energy loss is taken out of the plasma. We also put Bohm thermal conduction loses with a power of P= energy in the plasma/τ_{bohm}. This is reported as a worst case scenario for dissipation in the literature. As the plasma configuration would be some sort of toroid, we used half the radius of the cavity as the distance over which the energy dissipates (not the cavity radius). This energy is also taken out of the plasma. We only took energy out, no particles. The plasma is assumed to be 50% deuterium, 50% tritium. We calculate the fusion power with the fusion cross section found in [4]. At maximum compression B=7 MGauss and the alpha particles gyroradius is 0.7 mm; much smaller than the plasma (~4 cm). All the alpha particle energy (0.2 x fusion energy) is added to the plasma energy. As the total fusion fuel energy (6000 MJ) is much larger than the yield (700 MJ), fuel depletion is neglected.

The assumptions about the plasma are highly simplistic and much more work would be required there. However, the fusion yields found are in line with what other MTF calculations indicate for energy in the plasma of 14 MJ and density around 1E20 cm^{-3}. After playing with the various parameters, we found gains of about 6. This is just sufficient for a fusion power plant giving a thermal to mechanical conversion of 33% and a re-circulated power equal to delivered power. Note that the energy going in the plasma is only 12% of the fluid energy, so the intrinsic gain is 50. The poor efficiency is due to the large quantity of fluid moving. At maximum compression, most of the energy goes into compressing the fluid around the plasma and only 12% goes into the plasma. This is an unfortunate result that is due to the method used. However, because of the low cost of the driver, this poor efficiency is acceptable. Higher gains may be possible for larger machines. For example a gain of 7.5 was obtained at 140 MJ input energy in the simulation. However, these calculations were undertaken to find the minimum size machine that gives a reasonable gain. We actually want to build such a machine, so keeping size to the minimum is important.

Wall mixing with the plasma may reduce gain and is a big concern for most MTF schemes. Little is known about this problem. Asymmetry is another big issue that may reduce the gain significantly below these predictions. An interesting effect may help to increase the gain. The energy lost by the plasma is assumed to disappear. In normal magnetic fusion, this energy goes into the wall and heats it. Wall loading is a problem and care is taken that the wall survives. Here x-rays of ~20 KeV and ions and electrons of similar energy will hit the wall at a power density of ~1E11 W/cm^2. This energy will penetrate only ~100 μm (for the x-rays and less for the charge particles) into the liquid lead. The wall will not survive this. It will explode. The fusion is quenching after maximum compression because the plasma pressure pushes the liquid away and the density drops. If the wall is exploding as it is receding, the cold expanding Li-Pb plasma at the surface of the liquid metal may slow down the density decrease of the fusing plasma, extend the burn time and increase the gain. In effect, some of the energy loses are not completely lost but may return to the system via the wall exploding and compressing the fusing plasma. Adding this effect to the simulation would be interesting but was not accomplished here.

Details of the simulation

First, the initial conditions are set. The center of the sphere is on the left. The cells are spherical shell. All cells density=density$_0$ (kg/m^3), pressure=0 Pa, temperature=600 C (The melting point of lead), speed=0 m/s, positions (m) of the cell boundaries are uniformly spread with a distance dx between each. The mass of each cell=density$_0$*4π/3(position(right border)^3-position(left border)^3).

We then calculate the acceleration of each cell boundary:

The acceleration is the pressure gradient divided by the density:

$$\text{Acceleration}=\frac{\text{pressure(left cell)-pressure(right cell)}}{\text{(distance between the center of the left and right cell)*density(boundary)}}$$

Position=position+speed*dt+0.5*acceleration*dt^2
Speed=speed+acceleration*dt

$$\text{Density(center)}=\frac{\text{mass of cell}}{4\pi/3(\text{position(right boundary)}^3-\text{position(left boundary)}^3}$$

This is calculated for all cells from left to right. The speed and position are not updated immediately. They are stored temporarily. This is to avoid calculating the acceleration with an updated value on the left and an old value on the right.

After all cells are calculated, the speed and position are updated to the new values. The pressure of each cell is calculated from the equation of state.

From [2,3]:

$$\varepsilon=(1-\text{density}_0/\text{density})$$
$$P=bCT+\varphi(\varepsilon)$$
$$E=P/b+\theta(\varepsilon)$$

P is the pressure, T is the temperature and E is the internal energy. b and C are some constants, for liquid lead:

$b=\Gamma/\nu_0$
C=113.9 J/kgK is the thermal capacity at constant volume
Γ=2.943 is the Gruneison parameter
$\nu_0=1/\rho_0$=9.395E-5 m^3/kg is the specific volume
ρ_0=10644 kg/m^3 is the density
K_0=3.657E10 Pa is the isothermal bulk modulus

$c_s=(K_0/\rho_0)^{1/2}$=1853 m/s

c_s is the zero pressure adiabatic bulk sound speed

If data is known from shock wave experiments one often finds that:

$$U=c_s+su$$

Where U is the shock velocity and u is the particles speed just after the shock. Best fit to the experimental data give the constant s. For lead s is found to be 1.47.

From these relations and the Hugoniot shock equations you get:

$$b\theta(\varepsilon) =-\rho_0 c_s^2\varepsilon(1-s\varepsilon)^{-2}(1-\Gamma\varepsilon/2)$$

The function $\varphi(\varepsilon)$ and $\theta(\varepsilon)$ are not independent, they are related [3] by:

$\varphi'(\varepsilon) = \Gamma\varphi(\varepsilon) - b\theta'(\varepsilon)$

Where the prime denotes derivative with respect to ε . We can calculate the derivative:

$b\theta'(\varepsilon) =-\rho_0 c_s^2(1-s\varepsilon)^{-2}[1-\Gamma\varepsilon+2s\varepsilon(1-s\varepsilon)^{-1}(1-\Gamma\varepsilon/2)]$

We find $\varphi(\varepsilon)$ by numerical integration. We used ε step of 0.001 and we started with $\varphi(0) = -600bC$ so that P=0 at T=600 Kelvin (melting point) and $\varepsilon=0$. When using the function $\varphi(\varepsilon)$ we use linear extrapolation between the points separated by 0.001. Up to $\varepsilon=0.67$, corresponding to a pressure of 30 Mbar, all is fine but for higher ε the function $\theta'(\varepsilon)$ diverges and then goes negative. I.E. the pressure decreases for further compression, clearly non-physical. When using very small plasma (that is not an optimum size for large gain) ε at maximum compression exceeds this limit, crashing the code. So we clamp $\theta'(\varepsilon) = \theta'(0.67)$ for $\varepsilon>0.67$. This is obviously not exact and may affect the result. But this way the pressure keeps increasing rapidly with ε and the code at least behaves properly. For the baseline 120 MJ calculation yielding a gain of 6 the maximum pressure is 4.7 Mbar and ε does not exceed 0.67.

$P(T, \varepsilon)$ obtained from these calculations was compared to the shock curve in [2] until all bugs were fixed and the results agreed with the curve. This $P(T, \varepsilon)$ is all important and all results are highly dependant on it, you have to get it right.

The internal energy E is also calculated and is used for energy accounting to make sure energy is conserved. Energy checks are powerful to detect bugs and gain confidence in the code.

To calculate the pressure, we must first find the temperature.

From [3] one gets for and isentropic compression (no loss of entropy, I.E. perfectly reversible)

$$T=T_0e^{(\Gamma\varepsilon)}$$

Using this T and ε calculated from the density, one gets the pressure for each cell.

When the wave breaks into the free surface, the fluid is shot out at high speed and goes into tension. As a fluid does not have tension strength, it will break into cavitations, forming many small bubbles and the density will go below density_0. As the vapor pressure is negligible compared to the GPa of pressure involved, I assume that P=0 for density less than density_0 ($\varepsilon <0$). I do let the density go lower, because of the bubble in the fluid. When the fluid moves towards the center it gets compressed by the spherical convergence, the bubbles collapse and the pressure goes up again when the $\text{density}=\text{density}_0$. Unfortunately, because of this simplistic approximation, the discontinuity in the pressure-density function around $\varepsilon=0$ produces spurious oscillations when this condition is crossed. A bit of damping keeps these under control. The literature describes more complex EOS for two phases (vapor-liquid) state. I decided to stick to this simple approximation and I believe that only small errors are introduced.

Then damping is added like so:

$$\text{Damped speed}=\frac{\text{speed+ damping*speed (left)+damping*speed(right)}}{\text{1+2*damping}}$$

This procedure smoothes the speed function. If damping =1 (way too much), it replaces the speed of each cell by the average of the 3 cells: (cell + left + right)/3. It is exactly equivalent to put a viscosity term proportional to the second spatial derivative of the speed distribution in the acceleration. I do it separately so I can keep track of the energy dissipated.

$$\text{Energy dissipated}=0.5*\text{ cell masse}*(\text{Damped Speed}^2-\text{speed before damping}^2)$$

Without dissipation: $T=T_0e^{(\Gamma\varepsilon)}$

So the cell gets compressed to that temperature but viscosity reduces speed so I add this lost energy to the temperature:

$$T=T_0e^{(\Gamma\varepsilon)}+\frac{\text{Energy dissipated}}{\text{C*cell mass}}$$

Where C is the heat capacity.

Then I find what would be the temperature at 0 pressure if I decompress isentropicaly:

$$T \text{ (at } P=0) = \left(T_0 e^{(\Gamma\varepsilon)} + \frac{\text{Energy dissipated}}{C^{*}\text{cell mass}} \right) e^{(-\Gamma\varepsilon)}$$

I used this new temperature as T_0.

This procedure is found to conserve energy and when shock occurs, the condition after the shock are equal to what is obtained from the Hugoniot shock equations.

If damping is very small (<1E-4) doing this procedure leads to instability probably due to round off errors because of the very small energy losses. So I do not adjust the temperature for energy loses when dissipation <1E-4. This leads to total energy errors of less than 1 %. In the code I do no apply the damping at every time step, but only so often. This is to make sure that the energy dissipated is big enough to avoid round-off errors. The damping quoted here is for an equivalent per time step damping (actual damping/number of time step between applying).

Note that with positive speed curvature you borrow energy from temperature to speed up the cell in contradiction with thermodynamics. In a shock, the energy gained in the positive curvature at the beginning of the shock is much less than the energy lost in the negative curvature at the top (because the speeds are higher at the top), so it seems to work. But care must be taken to avoid big damping when a long positive curvature exists (E.G. when the cells approach the center and accelerate hard).

This damping tuning is not very satisfactory and must be done very carefully to avoid unphysical situation. The fusion yield decreases rapidly with increased damping so it is good to keep the damping as small as possible without the code going numerically unstable. That is why a bit of spurious oscillation is visible in the code. They can be damped, put at the cost of a lower yield.

I also tried an apparently better method. The total energy in each cell is calculated by the time integral of PdV at the cell's boundaries. You subtract the kinetic energy from this total energy to get the internal energy.

Using $P=bE-b\theta(\varepsilon)$ you get the pressure from the internal energy. This was found to be numerically unstable. Unfortunate, as it would not require the integration of $\theta(\varepsilon)$ to find $\varphi(\varepsilon)$ and adding the dissipated energy to the internal energy.

The total energy given by the integral of PdV is, however compared with the kinetic energy plus the internal energy. It is found to be equal for all the cells; a satisfactory result.

We then deal with the limit conditions. We use open ended conditions.

On the right end (the outside of the sphere):

Speed(past the end) = speed(end)+(speed(end)-speed(end-1))

You extrapolate the speed past the last cell with the same slope as the two last cells.
You do the same for the positions.

Pressure (past the end) = P(t)

Where P(t) is the applied pressure on the system. Care must be taken to always use smooth functions so as not to generate spurious shocks. We use pulses with leading edge and trailing edge of the form $P=P_0e^{-(t-tmax)2/t02}$. We choose $t_{max}>3t_0$ to avoid glitches at the beginning of the simulation. These small shocks pick up energy as the pulse travels and can grow large, spoiling the results. We check that the total energy in the system = integral of PdV= P(t)*4π*position (end)^2*speed(end)*dt on the outside of the sphere from this external pressure.

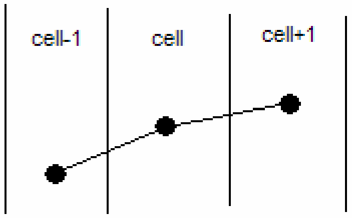
In the inside (facing the plasma) we use similar conditions of extrapolating with the same slope to the cell past the edge. Except for the pressure:

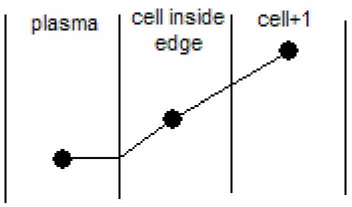
Pressure (inside) = plasma pressure

The increased energy in the plasma = integral PdV =
Plasma pressure*4π*position(cell on the inside edge)^2*speed(cell on the inside edge)*dt

This most equal the energy lost by the fluid. Also, taking out fusion gain, radiation losses and Bohm losses, this most equal the adiabatic result: Energy in plasma=E_0/r^2, which it also does.

We detected a small issue there. Between normal cells within the fluid we find the pressure gradient at the boundary from a linear extrapolation between the pressures of the cells (defined in the center of the cell).





For the inside edge it is a big plasma not a small fluid cell with a steep gradient. So the pressure gradient accelerating the left boundary of the edge cell is the pressure of the last cell minus the plasma pressure divided by half the last cell thickness. This apparently small detail was dissipating a lot of energy (30%), reducing the speed of the collapse.

We then check for cell doubling if a cell gets more than twice as thick as original thickness. We add a boundary in the middle of the cell that is too thick. We put the same speed as the original cell for this new boundary. The mass of the two new cells defined by this new boundary are half the original cell.

We then calculate the plasma parameters.

We assume a totally ionized plasma with Z=1 and ion temperature equal to electron temperature;

$n=n_e=n_i=n_0$
$T=T_e=T_i=T_0$
So $P=n_e kT+n_i kT=2nkT$
And $E=3/2N_e kT+3/2N_i kT=3Nkt$
$N=nV$

Where n is the plasma density, V is the original volume, k is the Boltzman constant. N is the total number of electrons (or ions) in the plasma, E is the energy in the plasma. The code uses the temperature T in keV.

During the run we calculate the compression power dumped into the plasma =

Plasma Pressure*4π speed(inside edge cell)*position(inside edge cell)^2.

We also add 20% of the fusion power (from the alpha particles) and subtract Bremsstrahlung radiation losses and Bohm thermal losses.

For each time step:

Energy in plasma = energy in plasma + compression power*dt + 0.2*fusion power*dt - Bremsstrahlung power*dt - Bohm losses power*dt

The new density is: $$n=n_0*\frac{\text{position(initial inside edge)}^3}{\text{position(inside edge)}^3}$$

The new temperature: $$T=\frac{\text{Energy in plasma}}{3*Nk}$$

The new pressure: $P=2nkT$

The fusion power density is [4]: $n_d\, n_t*\sigma v(T)*17.6\ \text{MeV}$
$2.8E\text{-}12*\sigma v(T)*\ n^2/4\ \ \text{W/cm}^3$

We multiply this by the volume 1E6*4/3πposition(inside edge)^3 to get fusion power

The factor 1E6 is because the cell position is in m and the power density is W/cm^3

$\sigma v(T)$ is [4]: $$\frac{3.68E\text{-}12}{T^{2/3}} \exp(-19.2/T^{1/3}) \quad cm^3/s$$

With T in keV for T<25 keV

In most simulations (including our baseline calculation) T is less than 25 keV. For case where T>25 keV we clip the cross section at 9E-16, the value at the top of the curve. This overestimates the fusion yield somewhat.

The Bremsstrahlung losses are given by [4]:

$$5.3E\text{-}31\ n^2T^{1/2}\ W/cm^3$$

Again we must multiply by the volume to get the total power lost.

The Bohm thermal losses are [5]:

Power lost=Energy in plasma/τ_{bohm} W

$\tau_{bohm}=a^2/\chi_{bohm}$

a is the distance over which the energy flows out. Because the plasma is a toroid inside a cavity of radius r, a=r/2 not r.

χ_{bohm} is the Bohm thermal diffusivity. We choose Bohm diffusion in place of the smaller classical diffusion. Real plasmas are found to diffuse faster than classical but generally slower then Bohm. So this is a conservative assumption.

$\chi_{bohm}=r_i\lambda\nu_{ii}/16$

r_i is the ion gyroradius in the magnetic field
λ is the electron mean free path
ν_{ii} is the ion-ion collision frequency

$r_i=7320\ T^{1/2}/B$ in cm with T in keV and B in Tesla for deuterium ion

$\lambda=1/n\sigma$ in cm

$\sigma=5.8E\text{-}20\ \log(\Lambda)/T^2$ is the coulomb electron collision cross section in cm^2, T in keV

$\Lambda=4.9E14\ T^{3/2}/n^{1/2}$ is the plasma parameter with no unit, T in keV, n in cm^{-3}

$\nu_{ii}=\nu_{ee}/63$ for deuterium ion

$\nu_{ee}=v_e/\lambda$ is the electron-electron collision frequency in s^{-1}

v_e=1.27E9 $T^{1/2}$ is the electron thermal velocity in cm/s, T in keV

We integrate the fusion power to give the total fusion energy yield.

We also integrate the compression energy up to maximum compression to measure how much compression energy goes in the plasma.

Results

We first investigate the transfer of energy from the outside edge of the sphere to the plasma interface near the center. We first launch a gentle wave of P=0.5E9 $e^{(-t2/(20us)2)}$ with very little damping of 3.3E-5 (just enough so it remains stable) (link: sima.exe). The sphere outside radius is 1 meter and the inside radius where the plasma will be injected is 20 cm. The bottom line is the density; the top line is the speed (negative speed towards the left). For the density the horizontal line is at the initial density of 10.8. The middle horizontal line is speed=0 and the full scale is +/- 200 m/s. The red bar on the left is kinetic energy, the green bar is internal energy, and the blue bar is the energy in the plasma. The full scale to the top is an energy of 10 MJ. As can be seen the wave is nice, half the energy is in kinetic energy and half in internal energy. When the wave breaks out, all the compression energy goes into kinetic energy. The wave peak speed goes almost as 1/r as it should. Some energy (20%) is lost in some fluid going right after the wave passage. This is because the wave goes into lighter and lighter cells (because they have same thickness but smaller radius). There is a small impedance mismatch causing some reflected energy. Using smaller cell size reduces this problem to acceptable levels but slows down the calculation. This is a purely numerical effect and is not expected for the real case. One can also see a bit of leading edge steepening due to the non-linearity of the fluid.

The maximum launching pressure is 2E9 Pa. At that pressure and damping of 3.3E-5, the wave develops strong oscillation when the shock wants to develop but cannot shed its energy (link: simb.exe). The scale of the energy bar is now 100 MJ at the top of the rectangle and the speed scale is +/- 1 km/s. Note that the energy is dissipated by this small damping because of the wild oscillation. There is no attempt at turning the dissipated energy into thermal energy for this small damping. Putting a damping of 0.001, we observe a nice transition to a shock (link: simc.exe). Notice how energy now moves from kinetic to internal but the total energy remain constant. It is nice to see that just by adding damping one can get the shock wave right. However, only ~80% of the energy makes it into the center,

Increasing pulse length to P=2E9$e^{(-t/40\mu s)2}$ delays the shock formation to just before the center and most of the energy makes it (link: simd.exe). The full scale for the energy bar is now 200 MJ. This is an important result, we can transmit large amounts of energy (100 MJ for this case) through the lead to the plasma with low loses. However, the peak power is limited (1 TW for this case) by the non-linearity of the fluid that forms a shock and dissipate energy if the power is too high.

The pressure for a metal on metal impact has a very fast rise time. The duration of the pulse is twice the transit time of the acoustic wave across the impacting piston. We launched a 100 MJ, 2E9 Pa square pulse with a fast 1 μs rise time (link: sime.exe). Because of the sharp edge, the shock starts right away and we lose 20% of our energy in transit; not good.

We will therefore need to make a longer rising edge. For that, a layer of soft material can be laminated on the piston and/or impacting steel so the pressure rise is more gradual. We need a rise time of ~44 μs at ~100 m/s= 4.4 mm thick. Alternatively a well chosen air pressure can be left between the pistons and impact steel in place of a vacuum. Bumps on the pistons and/or impact steel may also be used to blunt the edge of the pulse. The peak pressure is adjusted by the piston velocity, total pulse length and therefore energy by the piston thickness. A 2E9 Pa pulse of 80 μs FWHM deliver 100 MJ in the fluid from a 200 mm thick steel piston at 100 m/s. (density=8, sound speed 5 km/s in steel). At an operating temperature of 330 C (melting point of lead) typical of a steam power plant, the steam pressure is 1800 psi. At that pressure the piston can be accelerated to 100 m/s in a 1.3 meter long cylinder, a reasonable length.

We now adjust the parameters to get a fusion gain of 6 relative to the input energy. We found that a pressure pulse of $P=2E9e^{(-t/45\mu s)2}$ with the following parameters provides a gain of 6 (link: simf.exe):

Incoming pulse energy: 120 MJ
Initial plasma density: 1.25E17 cm^{-3}
Initial plasma temperature: 100 eV
Initial magnetic field: 7 Tesla
Initial plasma radius: 20 cm
Radial compression: 9.76
Energy transfer to plasma: 14 MJ
Fusion energy produced: 704 MJ
Energy gain: 5.9
Maximum fluid-plasma surface velocity: -2609 m/s
Peak plasma density: 1.16E20 cm^{-3}
Peak plasma temperature: 24.6 keV
Peak plasma pressure: 4.7 Mbar
Peak magnetic field: 666 Tesla
Confinement time (FWHM of plasma density): 6.93 μs

The simulation also gives plasma density (blue bell shape graph, 2E20 cm^{-3} full scale), temperature (green, 60 keV full scale) and fusion power (red, 100 TW full scale) as a function of time around peak compression. The dots are separated by 330 ns (1 μs/3 dots). The energy bar on the left has 300 MJ full scale. The graph changes scale at wave break-out. The speed scale is +/-1 km/s before and +/- 5 km/s after the scale changes. The black window where some numbers can be read is hidden under the front window. Just move the front window to read the results.

The peak plasma pressure achieved is 4.7 Mbar. That is in relatively good agreement with the stagnation pressure for a moving fluid of ρv^2=1 Mbar (using the lead density at maximum speed 14428 kg/m^3 and the maximum speed -2609 m/s). The pressure obtained is higher because there is still some convergence after the point of maximum speed. In a way similar to inertial confinement, the confinement time should be of the order of

r_{min}/c_s=10 μs, a correct order of magnitude. So the code produces realistic conditions. However, the exact fusion yield is a fast function of peek plasma conditions and therefore very sensitive to the exact hydrodynamic. Although the simulation likely produces a good qualitative view of what is going on, the exact fusion yield obtained is only indicative.

It is instructive to play with the input parameters and look at the fusion yield. Initial plasma temperature must be adjusted to obtain a final plasma temperature just enough to obtain ignition, then the temperature shoots up from the alpha particle heating. Too high an initial plasma temperature reduces the peak density and therefore fusion yield. If initial density is increased the energy to compress it to a given ratio (~10 seems to be the popular number in MTF research) also increases. Higher initial plasma density improves the gain but requires more energy. We decided to stop at a realistic (although quite high) initial density of ~1E17 cm^{-3} and total energy of ~100 MJ. The size of the initial plasma is also critical. A bigger plasma requires more energy to compress. The losses are also less for larger plasma. Like any other fusion scheme, bigger is better and this simulation just indicates the approximate size required for a fusion gain just enough to produce power. There is some trade off between plasma density and size. Too small a plasma does not couple so much of the fluid energy in the plasma. Too large a plasma takes too long to compress and the radiation losses cool it down too much.

Unfortunately only about 12% of the incoming energy makes it into the plasma. This 12% seems a robust result; many changes of the parameters aimed at improving this coupling did not produce significant changes. We therefore require an intrinsic fusion gain of 50 to get a total energy gain of 6. This forces us to use a bigger machine delivering more energy to the plasma to achieve these higher gains.

So we can see that a machine of 2 m diameter with an input energy of 120 MJ can give gains of interest. Those are reasonable numbers.

To get a feel for the severity of the problems if some of the wall material diffuses in the plasma we use a $Z_{effective}$ of 1.4 (2 time more Bremsstrahlung loses). This reduced the gain to 4.7. Note that the initial temperature needed to be increased to 130 eV to get that gain otherwise the plasma failed to ignite because the increased losses makes it too cold. The gain is quite sensitive to this problem so extreme care will be required to keep the plasma clean. Lead is pretty bad for a plasma facing material, high Z and low melting point. If required, it may be technically possible to have a thin layer of pure liquid lithium floating on the liquid lead-lithium to improve $Z_{effective}$. A freshly injected first wall of liquid lithium (Z=3) should have very little oxygen, water vapor, pump oil or other contaminant on its surface reducing plasma contamination.

Note that the initial plasma conditions are at wave break out. The wave takes a long time (500 μs) to get to the center. If the plasma is left for so long, it cools down. So in a machine the plasma would have to be injected just prior to wave break out. This requires the channel to remain open until that time. A 2D (r,z) simulation is required to investigate that in better detail. However a simple calculation is instructive. The speed of the fluid goes like 1/r as it moves towards the center. The time for the channel to close will therefore go like r. The simulation gives a closing time of 150 μs at r = diameter of initial plasma=20 cm. The time of closing will be the travel time of the wave=(R-r)/c_s plus the closing time; where R is the outside diameter of the tank and c_s is the speed of sound. For our numbers, the channel will close at 750 μs at r=R and at 595 μs at r=20 cm. So the channel will stay open long enough to shoot the plasma in it just prior to wave break-out. The exact shape of the collapsing tunnel can be adjusted by using a non-spherical system. The exact shape is adjusted so we have time to inject the plasma in the vortex tube just prior to wave break-out and the collapsing cavity trapping the plasma closes in axially and radially, providing 3D compression and therefore reducing the required convergence compared to a 2D compression.

We also have the major problem of the symmetric collapse of the vortex. Again a 2 D (r, θ) simulation would shed some light on this. Because the stagnation time is around 7 μs, a rough guess is that the wave must be controlled to 1 μs or so. Controlling the impact of ~100 kg pistons accelerated to 100 m/s to 1 μs requires controlling the trajectory to 100 μm, a difficult task. High performance linear translation stages with speed of 2 m/s and accuracy of 5 μm are commercially available. So it appears that with a suitable servo control system the required performance could be achieved. As the wave travels towards the center, the symmetry remains constant; there is no instability or smoothing effect. If the wave developed into a shock, there is some smoothing. If a dip exists in the shock front, the shock focuses there and accelerates, filling the dip. If a bump exists, the shock defocuses and slows down. This effect is well known. However, we want to limit the formation of shocks in the fluid so it is unlikely to help us a lot. Maybe some trade off between losing energy and smoothing asymmetry would be desirable. At break out, the liquid pressure goes to zero. It basically goes ballistic with no pressure applied and that is neutral. The plasma in front of the liquid is much less dense than the fluid and is accelerated by the fluid; a Rayleigh-Taylor (RT) stable situation. When the fluid velocity and pressure increases rapidly near the center, it is not clear what the stability is. Very near the center, the plasma pressure starts slowing down the fluid. This situation is RT unstable but only for a very short time. In the simulation, the initial radius is 20 cm, the point where the fluid starts slowing down is at r = 3.3 cm and the maximum compression occurs at 2 cm; not much motion to develop the instability. These symmetry issues and plasma-wall mixing are the biggest risk factor for this proposal.

We conclude that our system with a tank diameter of ~2 m and an input energy of ~120 MJ could provide interesting fusion gain. Such a system could be built at a reasonable cost of ~10 $M in about 3 years. (For a prototype with low repetition rate, no tritium re-breeding, no heat exchanger and no turbo-generator). The remarkably low cost of such a machine is because of the mechanical nature of the power driver. The concept also has very appealing nuclear engineering characteristics that could lead to a realistic commercial fusion power plant. The relatively small size (~50 MW electric) of the plant would reduce the time and cost of development.

References

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